

Axion-induced oscillations of cooperative electric field in a cosmic magneto-active plasma

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We consider one cosmological application of an axionic extension of the Maxwell-Vlasov theory, which describes axionically induced oscillatory regime in the state of global magnetic field evolving in the anisotropic expanding (early) universe. We show that the cooperative electric field in the relativistic plasma, being coupled to the pseudoscalar (axion) and global magnetic fields, plays the role of a regulator in this three-level system; in particular, the cooperative (Vlasov) electric field converts the regime of anomalous growth of the pseudoscalar field, caused by the axion-photon coupling at the inflationary epoch of the universe expansion, into an oscillatory regime with finite density of relic axions. We analyze solutions to the dispersion equations for the axionically induced cooperative oscillations of the electric field in the relativistic plasma.

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I. INTRODUCTION

The Vlasov concept of a cooperative macroscopic electromagnetic field in a collisionless plasma [1, 2] (or, equivalently, self-consistent, collective, average, mean field) is nowadays one of corner-stones of modern plasma theory (see, e.g., [3–6]). The Maxwell-Vlasov theory, based on this concept, deals in fact with a two-level self-regulating system: electrically charged plasma particles move under the influence of a macroscopic electromagnetic field and produce the cooperative electric current, which, in its turn, generates the self-consistent electromagnetic field. Thus, the ensemble of plasma particles controls itself by means of cooperative electromagnetic field. Master equations of the Maxwell-Vlasov theory include, respectively, two coupled sub-systems: electrodynamic and kinetic equations.

When we consider the Maxwell-Vlasov-axion model, we deal with a self-consistent theory of interaction between electromagnetic, pseudoscalar (axion) fields and a multi-component plasma. This physical system can be indicated as the three-level one, since in addition to plasma particles and photons it includes axions, i.e., hypothetical light massive pseudo-bosons, which appeared in the lexicon of High Energy Physics in the context of a strong CP-violation problem and spontaneous breaking of symmetry (see, e.g., [7–11]), and are considered as dark matter candidates (see, e.g., [12–14]). The contribution of the pseudoscalar field into the electrodynamic equations was discussed, first, by Ni [15]; the axion electrodynamics as an accomplished science appeared in the eighties of last century (see, e.g., [16, 17]). The corresponding master equations of the Maxwell-Vlasov-axion

model includes, first, equations of the axion electrodynamics (instead of the Faraday-Maxwell equations), second, the equation for the evolution of the pseudoscalar (axion) field, third, the kinetic equation for the plasma. This Maxwell-Vlasov-axion model is the corresponding part of the Einstein-Maxwell-Vlasov-axion model elaborated by authors in [18] and applied to cosmology in [19].

The axionic extension of Vlasov's idea prompted us to consider a cooperative pseudoscalar (axion) field, and to analyze Vlasov-type models of the axion-active plasma (we introduced this term in analogy with the magneto-active plasma). “Modus operandi” of the new axion cooperative field in plasma seems to be the following: axion field regulates the electromagnetic one via specific current-type term in the electrodynamic equations and (probably) form the axionic force in the kinetic equation; in its turn, the state of pseudoscalar (axion) field is predetermined by two pseudoscalar sources: of the electromagnetic origin and induced by plasma particles (see [18] for details).

We expect that this axionically extended model can be interesting for applications to cosmology and astrophysics. Indeed, the relic axions are assumed to form the cold dark matter, the contribution of which into the universe energy balance is estimated to be about 23% [20–22]. The mass density of the dark matter in the Solar system is estimated to be $\rho_{\text{(DM)}} \simeq 0.033 M_{\text{(Sun)}} \text{pc}^{-3}$ or in the natural units $\rho_{\text{(DM)}} \simeq 1.25 \text{ GeV} \cdot \text{cm}^{-3}$. Taking into account that the axion mass is assumed to belong to the interval $10^{-6} \text{ eV} \lesssim m_{\text{(axion)}} \lesssim 10^{-2} \text{ eV}$, the axion number density in the Earth vicinity can be (optimistically) estimated as $N_{\text{(A)}} \simeq 10^{11} - 10^{15} \text{ cm}^{-3}$. In addition, plasma is an important constituent of many objects and media in our universe, and the photons emitted, scattered or deflected by the plasma particles, propagate in the environment of axionic dark matter. Thus, models of the axion-photon coupling in a relativistic plasma can clarify some properties of the electromagnetic waves emitted by astrophysical sources and detected by astronomers.

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Axion-photon coupling is nowadays studied by many experimental groups; the most recent data concerning the modern status of these experiments can be found, e.g., in the reports of Collaborations abbreviated as PVLAS, GammeV, CAST, OSQAR, Q&A, BMV, ADMX, ALPS, XENON, EDELWEISS, CDMS, etc., published in the proceedings of last Patras Workshop on Axions and WIMPs [23].

In this paper we consider one application of the model established in [18]: the cooperative oscillations in the relativistic plasma, coupled to the axionic dark matter and to the global magnetic field, in the framework of the Bianchi-I cosmological model, the analysis of dispersion relations being the main purpose of the work.

II. THE MODEL

A. Kinetic equation

The first key element of the Maxwell-Vlasov-axion model is the relativistic kinetic equation

$$g^{is}p_s \left[\frac{\partial}{\partial x^i} + (\Gamma_{ik}^l p_l + e_{(a)} F_{ki}) \frac{\partial}{\partial p_k} \right] f_{(a)} = 0. \quad (1)$$

Here $f_{(a)}(x^i, p_k)$ is the 8-dimensional one-particle distribution function, which describes particles of a sort (a) with the rest mass $m_{(a)}$ and electric charge $e_{(a)}$; this function depends on four coordinates x^i and on the momentum four-covector p_k . The term $e_{(a)} F_{ki}^i p_i$ introduces the Lorentz force; F_{ik} is the Maxwell tensor, Γ_{ik}^l are the Christoffel symbols associated with the spacetime metric g_{ik} . Characteristic equations associated with the kinetic equation (1) are the following:

$$\begin{aligned} \frac{dp_k}{ds} - (\Gamma_{ik}^l p_l + e_{(a)} F_{ki}) \frac{dx^i}{ds} &= 0, \\ \frac{dx^i}{ds} &= \frac{1}{m_{(a)}} g^{ij} p_j. \end{aligned} \quad (2)$$

The four-interval ds is in this context a proper time for plasma particles.

B. Electrodynamic equations

The second key element of the model is the axionically extended Maxwell equations

$$\nabla_k F^{ik} = -F^{ik} \nabla_k \phi - I^i. \quad (3)$$

The term $F^{*ik} \equiv \frac{1}{2} \epsilon^{ikmn} F_{mn}$ describes the tensor dual to the Maxwell tensor F_{mn} ; $\epsilon^{ikmn} \equiv \frac{1}{\sqrt{-g}} E^{ikmn}$ is the Levi-Civita tensor, E^{ikmn} is the absolutely antisymmetric Levi-Civita symbol with $E^{0123}=1$. We use the dimensionless pseudoscalar field ϕ for description of axions. For such approach the axion-photon coupling constant appears in front of the kinetic term of the axion

field Lagrangian (see, e.g., [18] for details). Rescaling of this type is convenient for several problem of axion electrodynamics [24, 25], since two principal terms in the Lagrangian, $\frac{1}{4} F_{mn} F^{mn}$ and $\frac{1}{4} \phi F_{mn}^* F^{mn}$, have explicitly the same dimensionality.

The dual Maxwell tensor satisfies the condition

$$\nabla_k F^{*ik} = 0, \quad (4)$$

∇_k is the covariant derivative. The electric current four-vector I^i is considered to consist in a linear combination of first moments of the distribution functions $f_{(a)}(x^l, p_k)$

$$\begin{aligned} I^i &= \sum_{(a)} \frac{e_{(a)}}{m_{(a)}} \int \frac{d_4 P}{\sqrt{-g}} g^{ij} p_j f_{(a)}(x^l, p_k) \\ &\times \delta(\sqrt{g^{sn} p_s p_n} - m_{(a)}) \Theta(V^h p_h), \end{aligned} \quad (5)$$

where $d_4 P = dp_0 dp_1 dp_2 dp_3$ symbolizes the volume in the four-dimensional momentum space; the delta function guarantees the normalization property of the particle momentum, $g^{kl} p_k p_l = m^2$; the Heaviside function $\Theta(V^h p_h)$ rejects negative values of energy, V^h is the velocity four-vector of the system as a whole; g is the determinant of the tensor g_{ik} .

C. Equation for the pseudoscalar (axion) field

The third element of the model is the master equation for the pseudoscalar field ϕ :

$$[\nabla_m \nabla^m + m_{(A)}^2] \phi = \frac{1}{4\Psi_0^2} F_{mn}^* F^{mn}. \quad (6)$$

Here ϕ is a dimensionless quantity. The parameter $m_{(A)}$ is defined as $m_{(A)} = \frac{m_{(\text{axion})} c}{\hbar}$, where $m_{(\text{axion})}$ is the axion mass (let us remind that we use the natural units with $\hbar=1$, $c=1$). Estimations of the axion mass give the interval $10^{-6} \text{ eV} \lesssim m_{(\text{axion})} \lesssim 10^{-2} \text{ eV}$. The parameter $\frac{1}{\Psi_0}$ is the coupling constant of the axion-photon interaction; its estimations give $10^5 \text{ GeV} \lesssim \Psi_0 \lesssim 10^{12} \text{ GeV}$ (see, e.g., [23]).

D. Gravity field description

In the Maxwell-Vlasov-axion model we consider the gravitational field to be given and the space-time background to be described by the metric

$$ds^2 = dt^2 - a^2(t) [(dx^1)^2 + (dx^2)^2] - c^2(t) (dx^3)^2, \quad (7)$$

which relates to the well-known Bianchi-I anisotropic homogeneous cosmological model with local rotational isotropy. We consider the scale factors $a(t)$ and $c(t)$ to be known functions of cosmological time t . Also, we assume that the pseudoscalar, electric and magnetic fields inherit the spacetime symmetry: they depend on time only, as

well as, the electric and magnetic fields are directed along the anisotropy axis, i.e., $B^3(t) \neq 0$ and $E^3(t) \neq 0$ only. Let us stress that in this work we do not consider the electromagnetic and axion perturbations depending (generally) on all four coordinates; we study the oscillations only, i.e., we restrict ourselves by the solutions depending on time only. General analysis of the dispersion relations in the axion-magneto-active plasma is planned to be fulfilled in a separate paper.

III. SOLUTIONS TO THE MASTER EQUATIONS IN THE FRAMEWORK OF BIANCHI-I MODEL

A. Solution to the Vlasov equation

The Vlasov equation (1) is the homogeneous differential equation of the first order in partial derivatives, thus, the distribution function $f_{(a)}$ is arbitrary function of seven integrals of motion. One of the integrals, is clearly, the quadratic quantity $C_0 = g^{ik} p_i p_k$. For other integrals we have to fix the structure of the Maxwell tensor F_{ik} ; we assume that it contains two non-vanishing components only:

$$F_{12} = -a^2(t)c(t)B^3, \quad F_{30} = -c^2(t)E^3 = -\frac{dA_3}{dt}. \quad (8)$$

Here $A_3(t)$ is the longitudinal component of the electromagnetic potential four-vector. As for the component F_{12} , it is clear from (4) that it is a constant:

$$\nabla_k F^{*ik} = 0 \rightarrow \frac{d}{dt}[E^{i012} F_{12}] = 0 \rightarrow F_{12} = \text{const}. \quad (9)$$

Keeping in mind the characteristic equation $m_{(a)} \frac{dt}{ds} = p_0$, we readily obtain the so-called longitudinal integral of motion as follows:

$$dp_3 = e_{(a)} F_{30} dt \rightarrow C_{||} = p_3 + e_{(a)} A_3. \quad (10)$$

Two transversal integrals of motion can be found from the characteristic equations

$$\frac{dp_2}{p_1} = -\frac{dp_1}{p_2} = \frac{e_{(a)} F_{12} dt}{p_0(t) a^2(t)}. \quad (11)$$

The solution of (11) is known to have the form

$$p_1 = C_{\perp} \cos \Phi(t), \quad p_2 = C_{\perp} \sin \Phi(t), \quad (12)$$

$$\Phi(t) = \Phi(0) + e_{(a)} F_{12} \int_0^t \frac{d\tau}{p_0(\tau) a^2(\tau)}. \quad (13)$$

Clearly, we recover two well-known integrals of motion

$$C_{\perp}^2 = p_{\perp}^2 \equiv p_1^2 + p_2^2, \quad \Phi(0) = \arctan \frac{p_2}{p_1} - e_{(a)} F_{12} \int_0^t \frac{d\tau}{p_0(\tau) a^2(\tau)}. \quad (14)$$

The component p_0 of the particle momentum four-vector, which we need to calculate $\Phi(t)$, can be found from the quadratic integral of motion $g^{ij} p_i p_j = m_{(a)}^2 = C_0$:

$$p_0(t) = p^0(t) = \sqrt{m_{(a)}^2 + \frac{C_{\perp}^2}{a^2(t)} + \frac{[C_{||} - e_{(a)} A_3(t)]^2}{c^2(t)}}. \quad (15)$$

Other three integrals of motion are

$$C_4 = x^1(t) + \frac{C_{\perp}}{e_{(a)} F_{12}} \sin \Phi(t),$$

$$C_5 = x^2(t) - \frac{C_{\perp}}{e_{(a)} F_{12}} \cos \Phi(t),$$

$$C_6 = x^3(t) + \int_0^t [C_{||} - e_{(a)} A_3(\tau)] \frac{d\tau}{p^0(\tau) c^2(\tau)}. \quad (16)$$

Generally, the distribution function, which satisfies kinetic equation (1) can be reconstructed as an arbitrary function of seven integrals of motion: C_0 , C_{\perp} , $\Phi(0)$, $C_{||}$, C_4 , C_5 and C_6 . Taking into account that the spacetime is anisotropic and homogeneous, we require that all the macroscopic moments of the distribution function

$$T_{i_1 \dots i_s}(x) \equiv \int \frac{d_4 P}{\sqrt{-g}} f(x^i, p_k) p_{i_1} \dots p_{i_s} \quad (17)$$

inherit this symmetry, i.e., the Lie derivatives of the tensors $T_{i_1 \dots i_s}(x)$ (with arbitrary s) calculated along the Killing vectors ξ_{α} are equal to zero [27]:

$$\begin{aligned} \mathcal{L}_{\xi_{\alpha}} T_{i_1 \dots i_s} &= \xi_{\alpha}^l \partial_l T_{i_1 \dots i_s} \\ &+ T_{l \dots i_s} \partial_{i_1} \xi_{\alpha}^l + \dots + T_{i_1 \dots l} \partial_{i_s} \xi_{\alpha}^l = 0. \end{aligned} \quad (18)$$

As was shown in [28], the relationships (18) are satisfied for arbitrary s , when the distribution function satisfies the following equations:

$$\xi_{\alpha}^k \frac{\partial f}{\partial x^k} + (\partial_k \xi_{\alpha}^i) p^k \frac{\partial f}{\partial p^i} = 0. \quad (19)$$

The parameter α indicates the Killing vectors, which describe the spacetime symmetry; in the Bianchi-I model with local rotational symmetry we deal with four Killing vectors

$$\xi_1^k = \delta_1^k, \quad \xi_2^k = \delta_2^k, \quad \xi_3^k = \delta_3^k, \quad \xi_4^k = x^1 \delta_2^k - x^2 \delta_1^k. \quad (20)$$

For first three Killing vectors the relationships (19) yield

$$\frac{\partial f}{\partial x^1} = \frac{\partial f}{\partial x^2} = \frac{\partial f}{\partial x^3} = 0, \quad (21)$$

i.e., the distribution function does not depend on spatial coordinates x^1 , x^2 and x^3 . The fourth Killing vector provides the condition

$$p^1 \frac{\partial f}{\partial p^2} - p^2 \frac{\partial f}{\partial p^1} = 0, \quad (22)$$

which relates to the local rotational symmetry of the model. The conditions (21) require that the integrals C_4, C_5, C_6 (see (16)) can not appear as the arguments of the distribution function. The condition (22) holds, when p_1 and p_2 enter the distribution function only as $p_2^2 + p_3^2$. Thus, the distribution function has to include only three integrals of motion from seven, and should have the form

$$f_{(a)}(x^l, p_k) = f_{(a)}^{(0)}(C_\perp^2, C_\parallel^2) \delta(\sqrt{C_0} - m_{(a)}). \quad (23)$$

Here $f_{(a)}^{(0)}(C_\perp^2, C_\parallel^2)$ is arbitrary function of two arguments. As for the macroscopic velocity four-vector V^h , as usual for the Bianchi-I type spacetime, we put $V^h = \delta_0^h$. Then the cooperative electric current I^i happens to be reduced to the following term: $I^i = \delta_3^i \cdot I^3$

$$I^3 = - \sum_{(a)} \frac{\pi e_{(a)}}{a^2(t) c^3(t)} \int_0^\infty dC_\perp^2 \int_{-\infty}^\infty dC_\parallel f_{(a)}^{(0)}(C_\perp^2, C_\parallel^2) \times \frac{[C_\parallel - e_{(a)} A_3(t)]}{\sqrt{m_{(a)}^2 + \frac{C_\perp^2}{a^2(t)} + \frac{[C_\parallel - e_{(a)} A_3(t)]^2}{c^2(t)}}}. \quad (24)$$

Clearly, $I^1 = I^2 = 0$ since $f_{(a)}^{(0)}$ is even function of p_1 and p_2 . The component I^0 vanishes since the plasma is considered to be electro-neutral. The component I^3 as a function of the potential $A_3(t)$ displays the following properties:

$$I^3(A_3 \rightarrow 0) \rightarrow 0, \quad I^3(A_3 \rightarrow \infty) \rightarrow 0. \quad (25)$$

Finally, when the energy $e_{(a)} A_3$ of the plasma particle in the electric field is much smaller than its rest mass, i.e., $|e_{(a)} A_3| \ll m_{(a)}$, one can simplify the current as follows:

$$c^2(t) I^3(A_3) \rightarrow \Omega_L^2 \cdot A_3(t), \quad (26)$$

where the term

$$\Omega_L^2 \equiv \sum_{(a)} \frac{\pi e_{(a)}^2}{a^2 c} \int_0^\infty dC_\perp^2 \int_{-\infty}^\infty dC_\parallel f_{(a)}^{(0)}(C_\perp^2, C_\parallel^2) \times \frac{\left[m_{(a)}^2 + \frac{C_\perp^2}{a^2(t)} \right]}{\left[m_{(a)}^2 + \frac{C_\perp^2}{a^2(t)} + \frac{C_\parallel^2}{c^2(t)} \right]^{3/2}}. \quad (27)$$

is a generalization of the well-known Langmuir frequency [3, 5] (indeed, in the non-relativistic limit $\Omega_L^2 = \sum_{(a)} \frac{4\pi e_{(a)}^2 \mathcal{N}_{(a)}}{m_{(a)}}$, where $\mathcal{N}_{(a)}$ is the particle number density).

B. Solutions to the equations of axion electrodynamics

The Maxwell equations (3) reduce now to one linear differential equation for the function $A_3(t)$:

$$\ddot{A}_3 + \dot{A}_3 \left(\frac{2\dot{a}}{a} - \frac{\dot{c}}{c} \right) + \Omega_L^2 A_3 = F_{12} \dot{\phi} \frac{c}{a^2}. \quad (28)$$

Here and below the dot denotes the derivative with respect to time. The corresponding equation for the axion field takes now the form:

$$\ddot{\phi} + \left(\frac{2\dot{a}}{a} + \frac{\dot{c}}{c} \right) \dot{\phi} + m_{(A)}^2 \phi = \dot{A}_3 \frac{F_{12}}{\Psi_0^2 a^2 c}. \quad (29)$$

We deal with a coupled system of equations, which displays three interesting features. First, when $F_{12} = 0$ these two equations decouple, and the appropriate solution for $A_3(t)$ with initial value $A_3(t_0) = 0$ is the trivial solution $A_3 \equiv 0$. In other words, in the absence of magnetic field there are no reasons for the longitudinal electric field production. Second, when $F_{12} \neq 0$, and $\dot{\phi} \neq 0$, the electric field is inevitable, since the solution $A_3(t) = 0$ is not admissible. Third, when plasma is absent, (28) gives

$$\dot{A}_3 = F_{12} \frac{c}{a^2} \phi(t), \quad (30)$$

(we put here $\phi(t_0) = 0$ for simplicity), and the equation (29) converts into

$$\ddot{\phi} + \left(\frac{2\dot{a}}{a} + \frac{\dot{c}}{c} \right) \dot{\phi} + \phi \left[m_{(A)}^2 - \frac{F_{12}^2}{\Psi_0^2 a^4} \right] = 0. \quad (31)$$

As it was shown in [24] this model describes anomalous growth of the pseudoscalar field in the interval of the cosmological time, when $m_{(A)}^2 < \frac{F_{12}^2}{\Psi_0^2 a^4(t)}$. The formula (30) explains one very important detail inherent in the model: under the influence of the axion field pure magnetic field transforms into the pair of parallel magnetic and electric fields. In the paper [25] we indicated such field configurations as longitudinal magneto-electric clusters. The presence of the term $\Omega_L^2 A_3$ in (28) changes the behavior of the model principally. We expect now the appearance of solutions of the oscillatory type, and the cooperative electromagnetic field in the Vlasov plasma plays here the key role. Indeed, when the axion field produces electric field from the magnetic one, this electric field tries to separate plasma particles with positive and negative charges. However, this procedure generates the response of the cooperative electric field in plasma due to the Vlasov mechanism. Since the cosmological evolution is a non-stationary process, the interaction of axion-induced and cooperative fields leads to oscillations in plasma. Let us describe this process in more details.

IV. DISPERSION RELATIONS

First of all, let us discuss the hierarchy of time scales, which appear in the model.

The first time scale is predetermined by the Universe expansion. When one deals with isotropic universe expansion, effectively this time scale can be estimated using the Hubble function $\tau_{\text{cosmo}}^{-1} \rightarrow \mathcal{H}(t) = \frac{\dot{a}}{a}$. When we deal with anisotropic cosmological model, we have to use

the maximal rate of expansion $\tau_{\text{cosmo}}^{-1} \rightarrow \max\{\frac{\dot{a}}{a}, \frac{\dot{c}}{c}\}$. At present the value of the parameter τ_{cosmo}^{-1} is of the order $\tau_{\text{cosmo}}^{-1} \simeq \mathcal{H}(t_0) \simeq 10^{-18} \text{ s}^{-1}$; in the recombination epoch it was of the order $\tau_{\text{cosmo}}^{-1} \rightarrow \mathcal{H}(t_R) \simeq 10^{-13} \text{ s}^{-1}$; in the early Universe, near the inflation epoch, when the anisotropy could be essential, one can use the estimation $\tau_{\text{cosmo}}^{-1} \rightarrow 10^{-2} \text{ s}^{-1}$.

The second time scale relates to the frequency associated with the reduced axion mass; clearly, the mass range $10^{-6} \text{ eV} \lesssim m_{(\text{axion})} \lesssim 10^{-2} \text{ eV}$ corresponds to the frequency range $10^9 \text{ s}^{-1} \lesssim m_{(\text{A})} \lesssim 10^{13} \text{ s}^{-1}$.

The third time scale corresponds to the Langmuir frequency; for instance, for the thermonuclear plasma the estimations yield $\Omega_L \simeq 10^{11} \text{ s}^{-1}$.

Finally, the magnetic field B and the axion-photon coupling constant $\frac{1}{\Psi_0}$ define the fourth time scale parameter $\frac{B}{\Psi_0}$; for the range $10^5 \text{ GeV} \lesssim \Psi_0 \lesssim 10^{12} \text{ GeV}$ and the magnetic field $B \rightarrow 1 \text{ G}$ this parameters belongs to the interval $10^{-6} \text{ s}^{-1} \lesssim \frac{B}{\Psi_0} \lesssim 1 \text{ s}^{-1}$; for the relativistic rapidly rotating neutron stars with $B \rightarrow 10^{12} \text{ G}$, the basic quantity $\frac{B}{\Psi_0}$ is about 10^{12} times bigger; for the interstellar plasma with $B \rightarrow 10^{-6} \text{ G}$ it is, correspondingly, about 10^6 times smaller.

In our consideration we assume that the quantity τ_{cosmo}^{-1} is much smaller than the quantities Ω_L , $m_{(\text{A})}$, $\frac{\dot{\phi}}{\phi}$, and $\frac{B}{\Psi_0}$; for sure, one can find a lot of cosmological epochs, for which our assumption could be valid. In other words, we assume that the process of plasma self-regulation is much more quick than the process of the universe expansion. Keeping in mind this restriction, we put below $a(t_0)$ and $c(t_0)$ into the master equations of axion electrodynamics instead of functions $a(t)$ and $c(t)$.

Then to analyze the model we can use the standard Fourier-Laplace transformation

$$\mathcal{A}(\Omega) = \int_{t_0}^{\infty} dt A_3(t) e^{i\Omega(t-t_0)}, \quad (32)$$

$$\varphi(\Omega) = \int_{t_0}^{\infty} dt \phi(t) e^{i\Omega(t-t_0)}. \quad (33)$$

The corresponding equations for the Fourier-Laplace images take the form

$$\begin{aligned} \mathcal{A}[-\Omega^2 + \Omega_L^2] + i\Omega \varphi F_{12} \frac{c(t_0)}{a^2(t_0)} &= j_1, \\ i\Omega \mathcal{A} \frac{F_{12}}{\Psi_0^2 a^2(t_0) c(t_0)} + \varphi[-\Omega^2 + m_{(\text{A})}^2] &= j_2, \end{aligned} \quad (34)$$

where

$$\begin{aligned} j_1 &\equiv \dot{A}_3(t_0) - i\Omega A_3(t_0) - F_{12} \frac{c(t_0)}{a^2(t_0)} \phi(t_0), \\ j_2 &\equiv \dot{\phi}(t_0) - i\Omega \phi(t_0) - A_3(t_0) \frac{F_{12}}{\Psi_0^2 a^2(t_0) c(t_0)}. \end{aligned} \quad (35)$$

Clearly, all the information about poles of the Fourier-Laplace images $\mathcal{A}(\Omega)$ and $\varphi(\Omega)$ is encoded in the determinant of the system (34)

$$\Delta = \Omega^4 - \Omega^2 \left(\Omega_L^2 + m_{(\text{A})}^2 - \frac{B^2}{\Psi_0^2} \right) + m_{(\text{A})}^2 \Omega_L^2, \quad (36)$$

where we introduced the constant $B^2 = \frac{F_{12}^2}{a^4(t_0)}$ for the sake of simplicity. Let us discuss, first, three interesting particular solutions of the dispersion equation $\Delta(\Omega) = 0$.

A. Three special solutions

1. Magnetic field is absent, $F_{12} = 0$

In this case the dispersion equation gives two decoupled solutions: first, $\Omega = \pm\Omega_L$, which describes pure plasma oscillations; second, $\Omega = \pm m_{(\text{A})}$, which relates to the axion induced oscillations. The same result can be obtained, when $\frac{1}{\Psi_0} = 0$, i.e., the axion-photon coupling constant is equal to zero.

2. Massless axions, $m_{(\text{A})} = 0$

In this case one obtains that the first (double) root is $\Omega = 0$, and other two are the following: $\Omega = \pm\sqrt{\Omega_L^2 - \Omega_B^2}$, where $\Omega_B^2 \equiv \frac{B^2}{\Psi_0^2}$. We deal with oscillations, when $\Omega_L^2 > \Omega_B^2$. When $\Omega_L^2 < \Omega_B^2$, i.e., the magnetic field is strong or/and the axion-photon coupling constant $\frac{1}{\Psi_0}$ is rather big, we obtain non-harmonic processes in plasma: one increasing mode and one decreasing mode exist.

3. Langmuir frequency is small

When $\Omega_L^2 \rightarrow 0$, we obtain the double solutions $\Omega = 0$ and the solutions $\Omega = \pm\sqrt{m_{(\text{A})}^2 - \Omega_B^2}$. Again, we deal with oscillations, when $m_{(\text{A})}^2 > \Omega_B^2$, and with non-harmonic process, when $m_{(\text{A})}^2 < \Omega_B^2$.

B. General case: $m_{(\text{A})} \neq 0$, $\Omega_L \neq 0$, $F_{12} \neq 0$, $\frac{1}{\Psi_0} \neq 0$

In order to facilitate the analysis of this general situation, let us introduce the combination frequencies:

$$\Omega_{\pm} \equiv \Omega_L \pm m_{(\text{A})}. \quad (37)$$

The solution of biquadratic equation $\Delta(\Omega) = 0$ can be written as

$$2\Omega^2 = \frac{1}{2}(\Omega_+^2 + \Omega_-^2) - \Omega_B^2 \pm \sqrt{(\Omega_+^2 - \Omega_B^2)(\Omega_-^2 - \Omega_B^2)}. \quad (38)$$

Let us classify these roots.

1. Two different pairs of real roots

There are two different branches of oscillations, when

$$\begin{cases} (\Omega_+^2 - \Omega_B^2)(\Omega_-^2 - \Omega_B^2) > 0, \\ \frac{1}{2}(\Omega_+^2 - \Omega_-^2) + (\Omega_-^2 - \Omega_B^2) > 0. \end{cases} \quad (39)$$

Clearly, it is possible, when $\Omega_B^2 < \Omega_-^2$, thus, the formula

$$\Omega = \pm \frac{1}{2} \left[\sqrt{\Omega_+^2 - \Omega_B^2} \pm \sqrt{\Omega_-^2 - \Omega_B^2} \right] \quad (40)$$

is well-defined. As an illustration, let us calculate the electric field $E(t) = -\dot{A}_3$, which relates to the solution (40) and to the initial data $\dot{A}_3(t_0) = 0$, $A_3(t_0) = 0$, $\phi(t_0) = 0$:

$$\begin{aligned} E(t) &= E_0 \sin \left[\frac{1}{2} \sqrt{\Omega_+^2 - \Omega_B^2} (t - t_0) \right] \\ &\times \sin \left[\frac{1}{2} \sqrt{\Omega_-^2 - \Omega_B^2} (t - t_0) \right], \end{aligned} \quad (41)$$

where

$$E_0 = \frac{2 \phi(t_0) m_{(A)}^2 F_{12} \frac{c(t_0)}{a^2(t_0)}}{\sqrt{(\Omega_+^2 - \Omega_B^2)(\Omega_-^2 - \Omega_B^2)}}. \quad (42)$$

Thus, the maximum value of the total electric field in plasma is proportional to the modulus of the starting value of the pseudoscalar field $|\phi(t_0)|$, to the square of its mass $m_{(A)}^2$, and to the modulus of the initial value of the magnetic field $|F_{12}|$.

2. Two coinciding pairs of real roots

The solution of this type is of the form

$$\Omega = \pm \sqrt{m_{(A)} \Omega_L}, \quad (43)$$

it is possible, when $\left| \frac{B}{\Psi_0} \right| = |\Omega_L - m_{(A)}|$. Since the corresponding pole in the function $\mathcal{A}(\Omega)$ is the double one the contribution of this pole is linear in the cosmological time t ; in other word this case relates to the resonance-type growth of the electric field oscillations.

3. Two pairs of pure imaginary roots

We deal with two different pairs of pure imaginary roots, when $|\Omega_B| > \Omega_+$. When $\Omega_L + m_{(A)} = \left| \frac{B}{\Psi_0} \right|$ the pairs coincide yielding $\Omega = \pm i \sqrt{m_{(A)} \Omega_L}$. Keeping in mind the Fourier-Laplace transformations (32), (33), we see that one of the modes is the damping one, while the another mode is increasing and is proportional to $e^{\sqrt{m_{(A)} \Omega_L} t}$.

4. Complex roots: $\Omega = \alpha + i\gamma$

This is the most wide class of solutions, it relates to the following requirements:

$$\Omega_-^2 < \Omega_B^2 < \Omega_+^2, \quad (44)$$

and describes two modes of quasi-oscillations: increasing mode and decreasing mode.

V. DISCUSSION

In the framework of the Maxwell-Vlasov-axion model we described oscillations of electric field in an axion-active relativistic plasma, which are assumed to be produced in the course of anisotropic expansion of early universe with global magnetic field. Let us shortly summarize the results of this analysis.

1. The physical mechanism of the cosmic electric field oscillations seems to be the following. During the anisotropic stage of expansion of the early universe a specific phase transition is predicted to take place, which produces strong magnetic field (see, e.g., the review [26]). The pseudoscalar (axion) field produced in early universe by the Peccei-Quinn mechanism [7] transforms this (pure) magnetic field into the magneto-electric field, the electric component being parallel to the magnetic one and proportional to the pseudoscalar field. Such magneto-electric configuration (indicated as a longitudinal cluster in [25]), in its turn, provokes the exponential growth of the axion number and then, we obtain again the anomalous growth of the electric field. This inflationary-type procedure can be stopped due to the gravity field evolution, when the isotropic phase of the universe expansion replaces the anisotropic one (see [24]). As we have shown in this paper, the growth of the axion number and of the electric field, respectively, can be stopped by a more simple mechanism, namely, by the cooperative the Vlasov field counteraction in the relativistic plasma. Indeed, the axionically induced electric field (this field has no electrically charged sources, it is generated due to the axion-photon interaction in the strong magnetic field) separates particles with negative and positive charges thus generating the cooperative electric field, which counteracts to the global electric field. Since the universe expands, the described process is non-stationary, and the generated cooperative electric field tends to be in an instantaneous balance with global axionically induced electric field.

2. The growth of the total electric field can be stopped, when, first, the electrodynamic system is in an pure oscillatory regime; second, when it decreases with time non-harmonically; third, when a quasi-periodic oscillations with damping take place. In order to clarify the possibilities of the mechanism described above, we have analyzed the dispersion relations for the evolution of the coupled pseudoscalar (axion) and electromagnetic fields. We have found that every damping mode of quasi-oscillations

is accompanied by the corresponding increasing mode, thus, the only one interesting case exists, namely, the pure oscillatory regime, for which the growth of the axion number and of the electric field happened to be stopped by the cooperative Vlasov field. These situations can be realized, when $\Omega_B^2 < \Omega^2$; two frequencies of such oscillations are of the form (40); the estimation of the maximum value of the electric field is given by the formula (42).

3. We indicated the studied plasma model as axion-active plasma, since its internal state is predetermined by the state of global electric field produced by the pseudoscalar (axion) field in the presence of non-stationary magnetic field. This term is considered to be an analog of the

term magneto-active plasma. In the nearest future we hope to consider the dispersion relations for the *waves* in the axion-magneto-active plasma by using the solutions obtained here as a zero-order approximation in the problem of kinetics of axion-electromagnetic perturbations in the relativistic plasma.

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